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Samuel Winiger, Henrik Singmann, and David Kellen

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Bias in Confidence: A Critical Test for Discrete-State Models of Change Detection

Samuel Winiger
University of Zurich

Henrik Singmann
University of Zurich and University of Warwick

David Kellen
Syracuse University


Ongoing discussions on the nature of storage in visual working memory have mostly focused on 2 theoretical accounts: On one hand we have a discrete-state account, postulating that information in working memory is supported with high fidelity for a limited number of discrete items by a given number of “slots,” with no information being retained beyond these. In contrast with this all-or-nothing view, we have a continuous account arguing that information can be degraded in a continuous manner, reflecting the amount of resources dedicated to each item. It turns out that the core tenets of this discrete-state account constrain the way individuals can express confidence in their judgments, excluding the possibility of biased confidence judgments. Importantly, these biased judgments are expected when assuming a continuous degradation of information. We report 2 studies showing that biased confidence judgments can be reliably observed, a behavioral signature that rejects a large number of discrete-state models. Finally, complementary modeling analyses support the notion of a mixture account, according to which memory-based confidence judgments (in contrast with guesses) are based on a comparison between graded, fallible representations, and response criteria.

Keywords: visual working memory, change detection, critical test, discrete-state models, confidence

Research in working memory is concerned with our ability to hold and maintain representations of information over a short amount of time. This ability is closely associated with key human faculties such as reasoning (Süß, Oberauer, Wittmann, Wilhelm, & Schulze, 2002) and text comprehension (Daneman & Merikle, 1996), and has predictive value in important domains such as academic achievement (e.g., Bayliss, Jarrold, Gunn, & Baddeley, 2003). In recent years, considerable efforts have been made in the study of working memory in the visual domain, with particular focus on its capacity and storage mode. At this point, it is well established that visual working memory (VWM) has limited capacity, in the sense that there is an upper limit in the amount of information that one can maintain in working memory at a given time (e.g., Cowan, 2001). There is, however, an ongoing discus-

sion concerning the way information can be stored. This discussion has focused mostly on two theoretical accounts: On one hand, we have discrete-state or slot models, which assume that items are either stored in memory with high precision (each is stored in a “slot”) or not at all (e.g., Luck & Vogel, 1997; Rouder et al., 2008; Zhang & Luck, 2008). On the other hand, we have continuous resource models, postulating that information can be degraded in a more graceful manner, with the quality of each representation in VWM being determined by the amount of resources dedicated to it (e.g., Bays & Husain, 2008; van den Berg, Shin, Chou, George, & Ma, 2012; Wilken & Ma, 2004).

Like in many other research domains, the comparison of models of VWM is often predicated on a quantification of their ability to fit the entire data coming from some experimental design. These fits are made possible through a number of auxiliary assumptions, some parametric (e.g., latent distributions are Gaussian), others more substantive (e.g., processes are selectively influenced by certain experimental manipulations). Despite its successful track record, this approach to model comparison raises a number of concerns (Birnbaum, 2011; Kellen, 2019): For instance, a violation of any of the auxiliary assumptions made is likely to compromise the conclusions of a model-comparison exercise. This possibility is nothing more than the famous Duhem-Quine thesis (Duhem, 1954; Quine, 1953): Consider a theory \mathcal{T} , that along with auxiliary assumptions \mathcal{A} , makes the observable prediction \mathcal{O} . The failure to observe \mathcal{O} (i.e., $\overline{\mathcal{O}}$ is observed instead) does not imply a rejection of \mathcal{T} , given that \mathcal{A} might be at fault. For example, in the context of response-time modeling, Jones and Dzhafarov (2014) showed that

 Samuel Winiger, Department of Psychology, University of Zurich; Henrik Singmann, Department of Psychology, University of Zurich, and Department of Psychology, University of Warwick; David Kellen, Department of Psychology, Syracuse University.

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Correspondence concerning this article should be addressed to David Kellen, Department of Psychology, Syracuse University, 430 Huntington Hall, Syracuse, NY 13244. E-mail: davekellen@gmail.com

the ability of diffusion and ballistic accumulator models to successfully describe people’s responses is entirely dependent on a number of auxiliary distributional assumptions. The critical role of \mathcal{A} is also reflected in the (necessary) tinkering that takes place during model development (for a discussion and examples, see Shiffrin & Nobel, 1997).

Also challenging is the fact that goodness-of-fit measures, even when corrected for model flexibility, do not necessarily privilege the portions of the data that are most informative from a theoretical standpoint. This issue has been discussed at length by prominent theoreticians such as Rozeboom (1970), who argued for the importance of determining the empirical support for each of the different formal propositions that constitute a theory, rather than looking at omnibus support measures (Rozeboom, 2008).¹ To illustrate the point being made here, let us consider a couple of notable observations coming from physics and biology: (1) clocks on satellites orbiting the Earth run differently from clocks on Earth (e.g., Burns et al., 2017), (2) there are humans with two blood types (i.e., blood-group chimeras; Dunsford et al., 1953). Both observations have important theoretical implications. But because they are very specific or rare, they are likely to be downplayed in any model-comparison exercise that considers the fit to the entire data on time measurement or human genetics, along with a premium on model parsimony. There is a real possibility that cruder models that cannot accommodate these specific observations might end up striking a better overall compromise between parsimony and fit. For instance, imagine a scenario in which one would make the case for ‘first-generation’ globalist models of memory, despite their inability to account for null list-strength effects (Ratcliff, Clark, & Shiffrin, 1990), on the grounds that they can account for a number of other effects in the literature at the time (ca. the 1990s) and are simpler than its competitors.²

Considering these concerns, it is important for researchers to also consider some of the alternatives available in their toolboxes. The goal of the present work is to do so by comparing models of VWM using a *critical-test approach* (e.g., Birnbaum, 2008, 2011; Kellen & Klauer, 2014, 2015; Stephens, Dunn, & Hayes, 2018). The idea behind it is simple: Identify a specific prediction that contrasts different families of models and restrict all testing efforts to it. The result is a strong inference that speaks directly to the theoretical commitments of each model (Platt, 1964). In some cases, the sets of permissible outcomes associated with different families of models, let us say $\mathcal{O}_{\mathcal{M}_1}$ and $\mathcal{O}_{\mathcal{M}_2}$, are mutually exclusive (e.g., Birnbaum, 2008). What this means is that the set of permissible outcomes for one family of models corresponds to the set of forbidden outcomes of a competing family and vice versa. In other cases, the permitted outcomes of one family are a subset of the permitted outcomes of another: For example, $\mathcal{O}_{\mathcal{M}_1} \subset \mathcal{O}_{\mathcal{M}_2}$. These differences have important implications in the way researchers engage in critical testing: In the first case, the only concern is to ensure that our experimental design will not yield observations at the boundary of the different mutually exclusive predictions (e.g., Birnbaum, 2008). In the second case, researchers need to place their efforts toward reliably observing outcomes that are outside the subset $\mathcal{O}_{\mathcal{M}_1}$ (for an excellent example, see Stephens et al., 2018). In either case, it is possible to dismiss broad families of models and shift our focus toward more promising options.

The remainder of this article is organized as follows: First, we discuss discrete-state and continuous models of VWM in the

context of one of the main experimental paradigms used to compare them, the change-detection task (e.g., Luck & Vogel, 1997; Rouder et al., 2008; Wilken & Ma, 2004). We then discuss the auxiliary assumptions that are typically made in this context and propose a new critical test that does not require them. At the locus of this test is the specific way these models handle confidence judgments and the possibility of biased confidence, which we define later. We then report two experiments (one of them preregistered) that show the presence of biased confidence judgments, which are forbidden by discrete-state models. For the sake of readability, we first discuss somewhat simplified versions of the discrete-state and continuous model, and later show that the results of the critical test also hold across a number of more complex model variants. Finally, we report complementary model fits showing that, among the remaining candidate accounts, there is support for a mixture account in which memory slots provide graded, fallible representations.

Continuous and Discrete-State Models of the Change-Detection Task

In each trial of the *change-detection task*, illustrated in Figure 1, participants study a distributed array of items (e.g., squares) that vary on a single dimension such as color. After a brief presentation and some delay, one item location is probed with a test item and participants judge whether the color of that item has changed relative to the previous presentation, responding either “same” or “change.” Thus, in any change-detection task there are at least two trial types, change trials (with correct response “change”) and same trials (with correct response “same”). In addition, researchers typically manipulate the number of squares presented in a given trial (e.g., Luck & Vogel, 1997). It is also common to manipulate the proportion of same and change trials across different test blocks (e.g., Donkin, Nosofsky, Gold, & Shiffrin, 2013; Donkin, Tran, & Nosofsky, 2014; Rouder et al., 2008). In a few cases, participants are also requested to indicate how confident they are in their responses (e.g., Ricker, Thiele, Swagman, & Rouder, 2017; Wilken & Ma, 2004; see also Rademaker, Tredway, & Tong, 2012; van den Berg, Yoo, & Ma, 2017).

According to the *discrete-state model* illustrated in the top row of Figure 2, the tested item is stored in memory with probability m . Because the item is stored with high precision, a correct response is always expected.³ With probability $1-m$, no information about the item is stored and a guess has to be made, with response “same” being made with probability g , and response “change” being made with probability $1-g$. The discrete-state model can be

¹ “... astute evidence appraisal focuses on select features of the hypothesis at issue with only secondary confidence adjustments, if any, in its remainder. Holistic acceptance/rejection is for amateurs.” (Rozeboom, 2008, p. 1123).

² For purely rhetorical purposes, let us assume without further discussion that more models such as REM (Retrieving Effectively from Memory; Shiffrin & Steyvers, 1997) are more complex or flexible.

³ Please note that this assumption of high precision is only plausible when applying the model to experimental designs where the stimuli are highly discriminable (e.g. distinct color changes in change trials, such as blue→red). As discussed later, our experiments were designed in order to make this assumption plausible. Later, we discuss the implications of relaxing this high-precision assumption (despite the experimental design) in different ways.

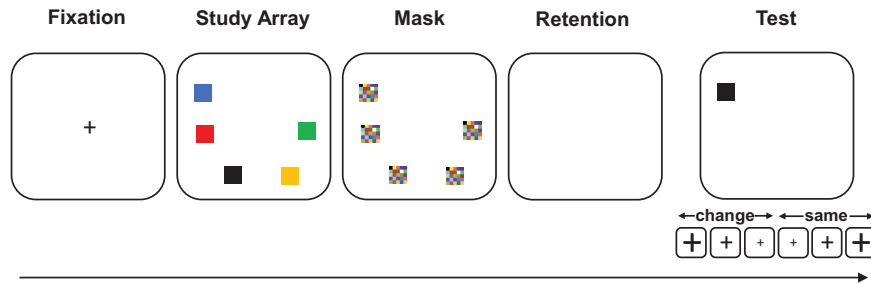


Figure 1. Illustration of the procedure of the change-detection task in visual working memory. Depicted is a change trial. Note that an eight-point confidence rating scale was used in the experiments reported here (the German verbal labels were *unterschiedlich* [change] and *gleich* [same]). Snapshots of one of the test trials can be found in the Open Science Framework project. See the online article for the color version of this figure.

extended to the case of confidence ratings by introducing confidence-mapping parameters for memory-based (ξ) and guessing-based responses (γ). These mapping parameters are traditionally assumed to be conditionally independent, such that their values do not depend on the value of m —all that matters is the discrete state one is in, not the probability of entering such state (for discussions, see Kellen & Klauer, 2015; Klauer & Kellen, 2010).

In the case of the *continuous resource model*, illustrated in the bottom row of Figure 2, the information available for a tested item can be represented as a sample from a latent-strength distribution, one for cases in which the item has changed, and another one for when it is the same. Both distributions are established on a latent memory-strength scale. Individuals judge the tested item by comparing its value with a response criterion τ , responding “same” when the value is larger than it, otherwise responding “change.” The continuous models can also be easily extended to accommodate confidence ratings. Specifically, one can introduce additional criteria τ , such that the intervals defined by them are mapped onto different levels of the confidence-rating scale (see Figure 2).

Response-Bias Manipulations and ROCs

The motivation behind the manipulating of the proportion of same and change trials is the belief that it selectively influences participants’ response biases, which are captured in each model by either the guessing probability g or the response criterion τ . Under this selective-influence assumption, we can derive clear predictions from both models about the way that the probability of “same” responses in same and change trials covaries across values of g or τ . These predictions are commonly referred to as the models’ receiver operating characteristic (ROC) functions (Green & Swets, 1966; Kellen & Klauer, 2018). We illustrate these ROCs in Figure 3. Binary-response ROCs have been used in a number of VWM studies (e.g., Donkin et al., 2013; Donkin et al., 2014; Rouder et al., 2008). Of course, it is entirely possible that response-bias manipulations also affect memory processes; that is, there is no selective influence (e.g., Balakrishnan, 1999; Van Zandt, 2000; see also Ashby, 1983; Diederich & Busemeyer, 2006; Ratcliff, 1981). If that is the case, then it is no longer possible to distinguish between both models based on the shape of the binary-response ROCs.

Confidence Ratings and ROCs

ROCs can also be constructed using confidence-ratings. Instead of being based on binary response proportion across different response biases, the ROC is constructed from the cumulative distributions across the confidence scale, from maximum-confidence “change” to maximum-confidence “same.” Wilken and Ma (2004) compared continuous and discrete-state models under the assumption that memory-based responses in the latter are deterministically mapped onto maximum-confidence judgments, which enforces the prediction of linear confidence-rating ROCs. The reliance on this assumption turns out to be fatal, given that it has been thoroughly dismissed by a number of theorists (Bröder & Schütz, 2009; Erdfelder & Buchner, 1998; Falmagne, 1985; Kellen & Klauer, 2014, 2015; Klauer & Kellen, 2010; Krantz, 1969; Luce, 1963; Malmberg, 2002; Province & Rouder, 2012). Allowing memory-based responses to include the range of the confidence scale that is consistent with the binary judgment enables the model to accommodate the curved confidence-rating ROCs that are typically observed.⁴

More recently, Ricker et al. (2017) proposed a new comparison relying on the notion of conditional independence, using a modified change-detection task. Instead of judging whether the color at test was the same presented before in that specific location (see Figure 1), participants were asked to choose between two colors. Also, they manipulated choice difficulty, such that the two colors presented were more or less distinct. Ricker et al. found that the confidence ratings obtained across choice difficulty levels did not conform with the assumption of conditional independence, therefore speaking against a discrete-state account. The problem with Ricker et al.’s conclusion is that it is questionable whether one could even begin to assume that conditional-independence holds in their experimental paradigm. As discussed by Kellen and Klauer (2015, p. 547), it is only reasonable to assume that conditional independence holds when dealing with experimental paradigms in which there are no external features informing the participant of the difficulty level of any given test trial. For example, in the domain of recognition memory, Kellen and Klauer (2015) focused

⁴ More specifically, the discrete-state model can predict piecewise linear ROCs that can capture the finite ROC data points collected in a given experiment.

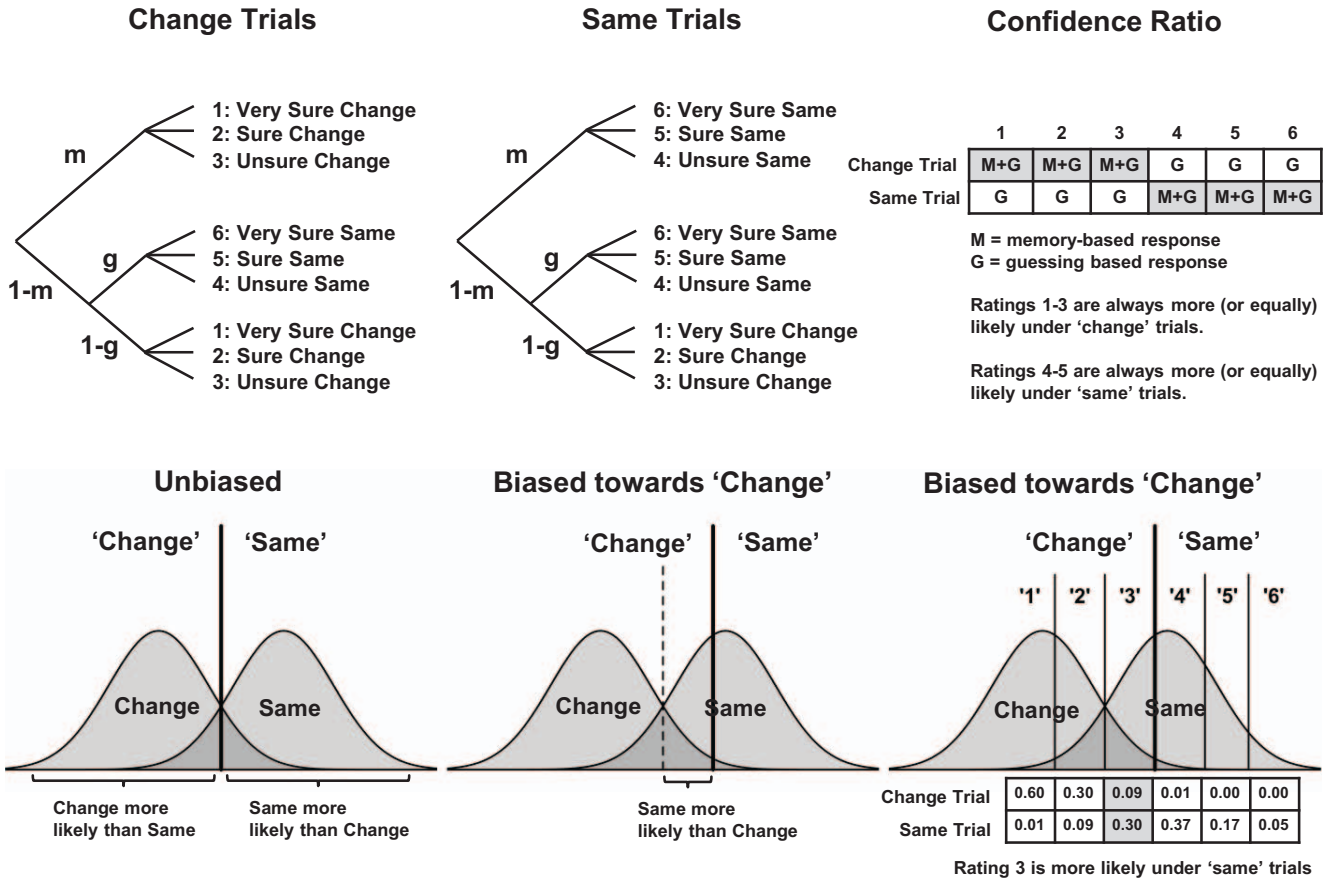


Figure 2. Top row: Discrete-state model for change and same trials, and the mixture components associated with each confidence level. In both trees, parameter m denotes the probability of an item being stored in working memory (in a slot), whereas parameter g denotes the probability of guessing “same.” The confidence levels are described by one of two components, with $M + G$ indicating that a given response confidence level in a given type of trial can be reached via both memory and guessing processes, and G indicating that it can only be reached via guessing. For clarity, the state-response mapping parameters ξ and γ (associated with memory and guessing states, respectively) are omitted. Bottom row: Illustration of the continuous resource model under unbiased and biased response criteria (criteria correspond to the vertical solid lines). The likelihood ratio associated with a given response corresponds to the average relative height of the two distributions within the region associated with that response. It is shown how the model can predict biased confidence judgments (e.g., “3: Unsure change” is more likely under same trials), something that the discrete-state model cannot do. Note that the Gaussian distributions and the equally spaced criteria are merely illustrative; they are not required for the discussed predictions to hold. For clarity, models for a six-point confidence rating scale are shown in the figure, whereas an eight-point confidence rating scale was used in the current experiment.

their testing of conditional-independence on study-strength manipulations that were not identifiable during the test phase. In each test trial, participants were shown a word, without any indication of whether it might have been studied once or thrice (see also Province & Rouder, 2012). In the case of Ricker et al.’s paradigm, these external features are obviously present, as they correspond to difference between the two colors.

A New Critical Test: Biased Confidence

The issues found with previous work using either binary-response or confidence-rating ROCs indicate the need for an alternative comparison approach that

1. does not assume selective influence in response-bias manipulations,
2. does not impose deterministic mappings on confidence judgments, and
3. does not compromise the assumption of conditional independence.

These requirements are achieved by the critical test proposed here, which is based on the notion of biased confidence judgments (Balakrishnan, 1999). In order to understand what this bias is, let us first consider the continuous resource model: As illustrated in

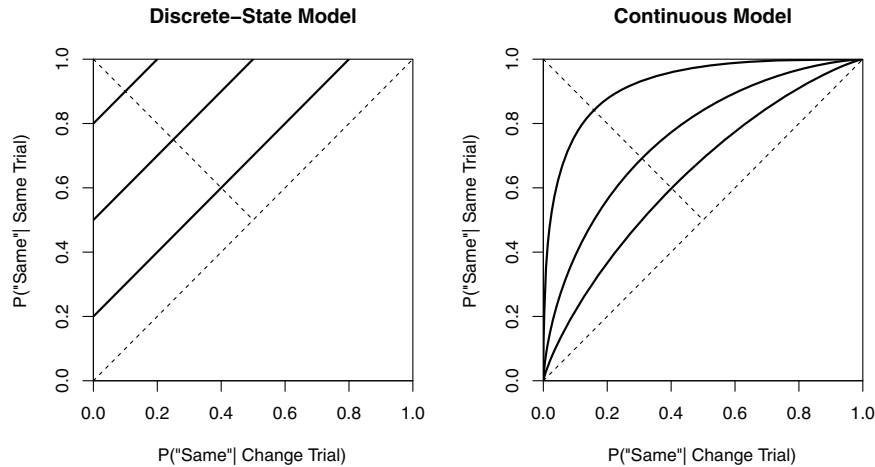


Figure 3. Illustration of different receiver operating characteristic (ROC) functions predicted by the discrete-state and continuous models for binary response data (i.e., no confidence rating responses).

Figure 2, confidence judgments result from the comparison between the latent-strength of a test item and a set of ordered confidence criteria τ . The position of each criterion relative to the latent distributions determines the likelihood of each item type given a certain response and confidence level. The introduction of a response bias, for example toward responding “change,” introduces the possibility of biased confidence judgments, especially when confidence is at a minimum, and minimum confidence covers a narrow range of strength values (Balakrishnan, 1999).⁵ This possibility follows from the continuous model’s core notion that confidence judgments are based on the segmentation of a latent-strength scale by confidence criteria. In the specific example given in Figure 2 (lower row, right panel), a minimum-confidence “change” response is more likely to occur in a same trial. This confidence level can be said to be biased as the respondent would improve their accuracy if they simply reassigned all their minimum-confidence “change” responses to minimum-confidence “same” instead (i.e., bundled all their minimum-confidence responses on the “same” side of the scale; see Balakrishnan, 1999).

In contrast, the discrete-state account precludes the possibility of biased confidence judgments. Confidence judgments result from the mapping of the different discrete-states onto a confidence scale, with the mapping of memory- and guessing-based responses being established by parameters ξ and γ respectively (for a discussion, see Klauer & Kellen, 2010). For instance, consider the probability of response “change” made with minimum confidence:

$$\begin{aligned} P(\text{“change”}_{\min} \mid \text{Change trial}) &= m \cdot \xi_{\min} + (1 - m) \cdot (1 - g) \cdot \gamma_{\min}, \\ P(\text{“change”}_{\min} \mid \text{Same trial}) &= (1 - m) \cdot (1 - g) \cdot \gamma_{\min}. \end{aligned}$$

Because the former probability cannot be smaller than the latter, confidence judgments cannot be biased (see Figure 2, top row, right panel). This inability to predict biased confidence judgments stems from the core notion within the discrete-state theory that, in the absence of stored information on the target stimulus, responses are invariably based on the same guessing process (i.e., conditional independence).

Continuous and discrete-state models can be compared by testing for the presence of biased confidence judgments. The discrete-state model only permits the inequalities

$P(\text{“change”}, \text{conf} = i \mid \text{Change trial}) \geq P(\text{“change”}, \text{conf} = i \mid \text{Same trial})$,
 $P(\text{“same”}, \text{conf} = i \mid \text{Same trial}) \geq P(\text{“same”}, \text{conf} = i \mid \text{Change trial})$,
 for all i among possible confidence levels, whereas the continuous model imposes no such constraint. What this means is that the set of permissible outcomes of the discrete-state model is a subset of the permissible outcomes of the continuous model. Our task then is to attempt to reliably observe cases in which these inequalities are violated; that is, try to obtain data belonging to the discrete-state model’s set of forbidden outcomes. Given our understanding of the continuous model, we expect the occurrence of forbidden outcomes to be most likely when (a) individuals show a clear bias toward one binary response, and when (b) individuals seldom make minimum-confidence judgments (see also Balakrishnan, 1999). These expectations are illustrated in Figure 2, which shows the occurrence of biased confidence judgments for minimum-confidence “change” judgments when there is a bias toward responding “change.” Given these expectations, we focus our analyses on minimum-confidence judgments.

Experiment 1

Method

We conducted a change-detection task experiment in which we attempted to observe biased confidence judgments. To ensure that the test was applied in conditions where items are expected to be stored with high fidelity (in line with the discrete-state model assumptions), we relied on highly dissimilar colors. Similar to previous studies (e.g., Rouder et al., 2008), biases in binary responses were encouraged by manipulating the proportion of change trials across blocks (75% vs. 25%).

Participants. A total of 44 participants took part in the experiment. Our plan was to collect at least 40 participants, in order to roughly match the sample sizes used in previous critical tests comparing continuous and discrete-state models (e.g., 45 partici-

⁵ In other words, the observation of a response bias is necessary but not sufficient for the observation of biased confidence judgments.

pants in Kellen & Klauer, 2014). We slightly overshot our intended target number as data collection was performed by a research assistant in a day-wise manner with as many participants as possible per day (i.e., on the second to last day of data collection we had not reached 40 participants, so we decided to continue for one more day).

The average age of our participants was 23.8 years, ranging from 18 to 29 ($SD = 3.7$). In exchange for their participation, participants received CHF 15 or course credit. Each session took about 50 min. All participants reported having normal or corrected-to-normal vision and normal color vision.

Stimuli and apparatus. Our stimuli and presentation generally followed Donkin et al. (2013). We used a set of 10 highly dissimilar colors (white, black, red, blue, green, yellow, orange, cyan, purple, and dark blue–green). These colors were taken from Table 5 of Donkin et al. (colors with suffix “-1”). Importantly, note that these colors are expected to yield “large” change trials (see Donkin et al., 2013, p. 891). The discrete-state model’s assumption that memory-based judgments are always accurate is assumed to be reasonable here (see Footnote 3; see also Nosofsky & Donkin, 2016). Stimuli were presented within a light gray rectangle of approximately 9.8×7.3 -degree visual angle. Stimuli were 0.75×0.75 degrees in size. Participants were seated approximately 60 cm away from the screen and no chin rest was used. The position of each stimulus was chosen randomly with the constraint of a minimal distance of two degrees from other stimuli and the screen center (measured from the center of the stimuli).

Procedure. The experiment was comprised of a practice block with 20 trials using a confidence-rating scale, followed by eight blocks with 52 trials each also using a confidence-rating scale, and one last block of binary-response trials. In the practice block and the final binary-response block, half of the trials were change trials, whereas in the remaining blocks using confidence ratings they were either 75% or 25% (four blocks with each proportion). The biased blocks were randomized, with the constraint that the same proportion of change trials did not occur more than twice in a row. Before each block, participants were informed about the percentages of change and same trials. In each trial, the percentages of change trials as well as the percentage of same trials were displayed in the labels shown above the confidence-rating scale (see the figure in the Open Science Framework [OSF] project).

Each trial followed the structure outlined in Figure 1: Each trial started with a fixation cross that was presented for 1,000 ms. An array of five square stimuli was then presented for 500 ms, followed by a blank screen for 500 ms. After the blank screen, a multicolored checkerboard-like mask was presented at each stimulus location for 500 ms. The test phase of each trial was self-paced: A test item was presented at a random stimulus location. Its color was either the same that was presented at the beginning of the trial at this location (same trial), or a color that was shown at the beginning of the trial, but at a different location (change trial). Participants were then asked to decide whether they are in a change trial or a same trial, while simultaneously stating their confidence by clicking on a response button on an eight-point confidence rating scale. For each half of the scale, confidence was represented by a plus sign (+) increasing in size from low to high confidence. A verbal label above each half of the scale clearly indicated the binary choice (i.e., “change” vs. “same”); the actual

German words used were *unterschiedlich* vs. *gleich*). Participants did not receive feedback on their performance.

Results

Manipulating the proportion of change trials succeeded in affecting individuals’ binary-response bias, although they were generally biased toward responding “change” across blocks. In blocks with 75% and 25% change trials, the average proportion of “change” responses was .78 and .40, respectively. In the last block with 50% change trials and binary judgments, the average proportion of “change” responses was .62. Overall, these proportions indicate a general bias toward “change” responses irrespective of experimental condition, although the base-rate manipulation appears to have produced an effect in the expected direction. This impression was corroborated using signal-detection bias measure c , which turned out to be more strict in 75% change condition ($M = 0.58$, $SD = 0.33$) than in the 25% change condition ($M = 0.17$, $SD = 0.39$; Wilcoxon’s $W = 941$, $p < .0001$). The left panel of Figure 4 shows the confidence-rating ROC functions obtained with the grouped confidence ratings, for each of the bias conditions. Note that the point obtained in the binary-response condition falls close to the ROCs. The ROCs take on concave and symmetrical shapes, as typically found in this type of data.⁶ Given these results, our hope to observe biased confidence judgments is limited to the 75% change condition. After all, this is the only condition in which we observe a strong response-bias effect, with minimum-confidence “change” responses being seldom made (see also Balakrishnan, 1999). For the sake of completeness, Figure 5 (first and second column) illustrates the results obtained in the 25% change condition, but we refrain any further discussion (for further details, see the OSF project).

As shown in Figure 5 (top row, third panel), the observed proportions of minimum-confidence “change” responses in the 75% change condition were generally greater in same trials, indicating the presence of a bias in confidence that is forbidden under the discrete-state model. Specifically, $P(\text{“change}_{\min}” | \text{Same trial})$ was greater/equal/smaller than $P(\text{“change}_{\min}” | \text{Change trial})$ in 68%, 11%, and 20% of the individual cases, respectively. At the group level, $P(\text{“change}_{\min}” | \text{Same trial})$ and $P(\text{“change}_{\min}” | \text{Change trial})$ were .12 and .07, respectively.

The individual minimum-confidence responses were fitted with a joint-binomial hierarchical-Bayesian model. In a Bayesian framework, the uncertainty regarding model parameters is represented via probability distributions, with prior distributions being updated into posterior distributions (via Bayes’s theorem) in light of the data. The key component of the joint binomial model adopted were the probabilities $P(\text{“change}_{\min}” | \text{Same trial})$ and $P(\text{“change}_{\min}” | \text{Change trial})$, whose difference was captured by an effect-size parameter δ with mean μ_{δ} (for details, see the OSF project). The range of predictions permitted under the discrete-state model correspond to a μ_{δ} that is greater or equal to zero, indicating that the minimum-confidence “change” responses are more likely in change trials than in same trials. The discrete-state model’s ‘forbidden region,’ which is expected under the continuous resource model, corresponds to a negative μ_{δ} , indicating that

⁶ As a reminder, the discrete-state model can accommodate curved confidence-rating ROC data.

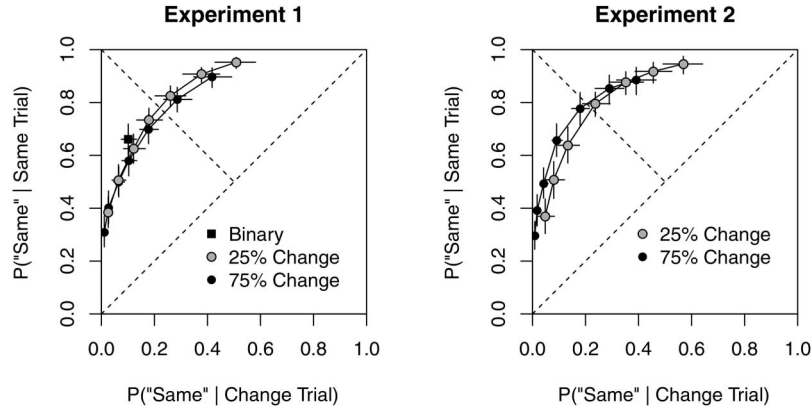


Figure 4. Circles show the receiver operating characteristic (ROC) functions obtained with the grouped data from both bias conditions. The square shows the results from the binary condition. The 95% confidence intervals associated with each point were obtained via nonparametric bootstrap.

such responses are more likely in same trials than in change trials. The visual inspection of the confidence-rating proportions in Figure 5 was corroborated by the joint binomial model’s posterior parameter estimates, which yielded a posterior mean of μ_{δ} of -0.19 , with a 95% credibility interval of $[-0.26, -0.13]$ (see Figure 5, top row, right-most panel).

The evidence for the discrete-state model’s forbidden predictions relative to its permitted predictions can be quantified by means of a Bayes factor (BF), which in the present case can be computed via the following ratio of posterior probabilities (Klugkist, Kato, & Hoijtink, 2005):

$$\text{BF} = \frac{P_{\text{post}}(\mu_{\delta} < 0)}{P_{\text{post}}(\mu_{\delta} > 0)}.$$

Values larger than 1 support the presence of biased confidence judgments, which are not permitted under the discrete-state model, whereas values between 0 and 1 indicate support for the absence of such bias. For the 75% change trials condition, we obtained a BF of roughly 60,000 (as none of the 60,000 μ_{δ} samples were larger than zero), indicating strong support for the presence of biased confidence ratings.

One potential objection to the analysis reported so far is that, whereas many of the minimum confidence responses fall into the ‘forbidden region’ of the discrete-state model, the magnitude of these violations might be comparatively small and still compatible with a discrete-state model when taking sampling variability into account. To corroborate the obtained BF, we conducted a *posterior-predictive test* (see Gelman & Shalizi, 2013; Klauer, 2010). Specifically, we generated one set of synthetic data, x_{synth} , per posterior parameter sample $\hat{\theta}_{\text{post}}$ from a joint-binomial model constrained to follow the predictions of the discrete-state model (i.e., μ_{δ} was constrained to be non-negative). We then computed the misfit of the true data, $\mathcal{L}(x_{\text{obs}}, \hat{\theta}_{\text{post}})$, as well as the misfit of the synthetic data, $\mathcal{L}(x_{\text{synth}}, \hat{\theta}_{\text{post}})$, according to the expectations derived from the posterior parameter sample. The latter can be understood as the expected amount of misfit if the only source of noise is sampling variability (and the true data generating process respects the constraints of the discrete-state model). We used these quantities to compute Bayesian p values by estimating the proba-

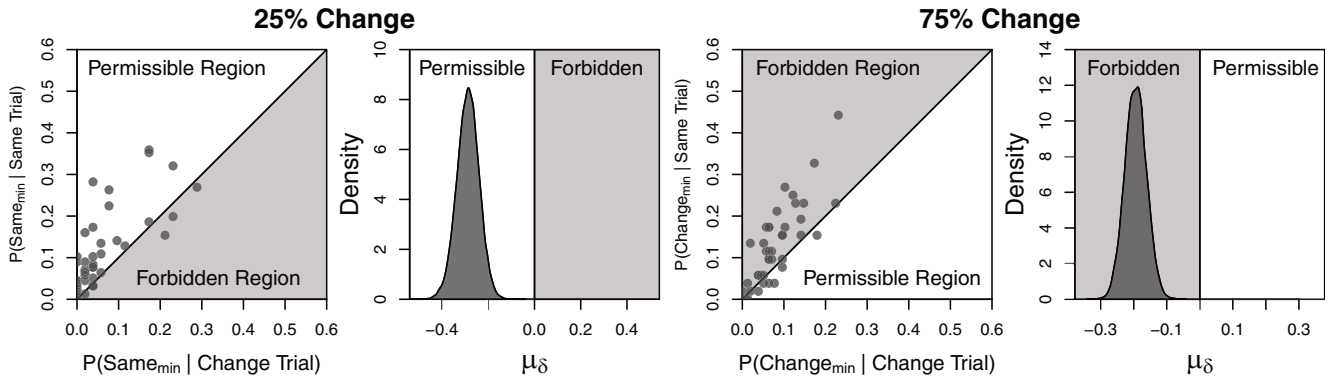
bility of the misfit of the real data being larger than of the synthetic data; that is, $P(\mathcal{L}(x_{\text{obs}}, \hat{\theta}_{\text{post}}) > \mathcal{L}(x_{\text{synth}}, \hat{\theta}_{\text{post}}))$. Small p values indicate a poor fit of the model to the data. We obtained a Bayesian p value of .003, which indicates that the observed data is very much at odds with the discrete-state model’s range of predictions, even when taking sampling variability into account. Importantly, when applying the same procedure to a joint-binomial model constrained to produce biased confidence judgments, the obtained Bayesian p value was .52, consistent with the fact that a confidence bias is generally present in the data.

Discussion

Although our results provide clear evidence against the discrete-state account, several concerns can be raised. First, our focus on minimum-confidence judgments might appear somewhat post hoc and even self-serving. After all, we focused our analyses on the minimum-confidence judgments coming from the 75% change condition, based on the fact that it was the only condition in which we observed a clear response-bias effect in the expected direction. Instead, shouldn’t one engage in joint evaluation of the evidence for biased confidence judgments across all condition? No, not at all. The first thing one should keep in mind is the nested (and thus asymmetric) relationship between the two theoretical accounts in the present experimental setup: One account includes the possibility of biased confidence ratings, whereas the other one does not. The failure to observe response biases or biased confidence ratings only indicates that both accounts are sufficient (i.e., the data are not diagnostic).⁷ In contrast, the observation of biased confidence judgments is highly diagnostic, as it cannot be accounted by a variety of discrete-state models (as will be shown later on). Given this state of affairs, our focus should be directed toward the

⁷ Note that sufficiency results are relevant when dealing with two theoretical accounts that differ in terms of their ontological complexity, such as single-process versus two-process accounts. Specifically, one can use them to make the case that there is no need to assume two-process accounts given that their single-process counterparts survived every attempt to reject them (for a discussion, see Stephens et al., 2018). However, this is not the case here.

Experiment 1



Experiment 2

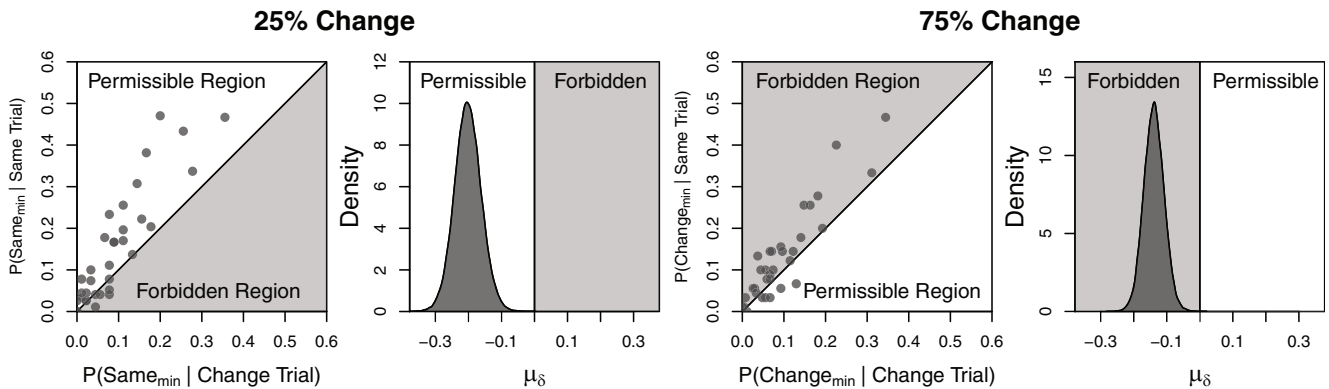


Figure 5. The scatterplots show the observed individual proportions of minimum confidence “same” responses (in the 25% change condition) or “change” responses (in the 75% change condition) in change and same trials. The density plots show the group-level posterior distribution of the difference parameter μ_δ . In all plots, the shaded areas correspond to the forbidden regions of the discrete-state model.

experimental conditions that seem most promising, namely the minimum-confidence judgments in a condition showing strong response biases (see Balakrishnan, 1999). The same strategy is found in other critical tests, where the focus is placed on individuals or groups showing specific preferences or successful study-strength effects (see Birnbaum, 2008; Kellen & Klauer, 2015; Kellen, Steiner, Davis-Stober, & Pappas, 2019). A joint analysis including nondiagnostic data would be a counterproductive move, akin to tempering one’s inferences coming from the observation of black swans by keeping tabs on all the white ones encountered along the way.

Another concern is that participants might not have fully understood the large differences between colors, which might have compromised their performance. This possibility is not implausible, especially given our failure to observe a bias toward “same” responses in the 25% change condition. Given these concerns, it is advisable to conduct a follow-up (preregistered) experiment in which we would try to minimize potential misunderstandings and would be more likely to observe a response bias in the 25% change condition.⁸

Experiment 2

Method

We implemented two major changes in our experimental design: First, we manipulated the proportion of change and same trials between subjects. The goal was to increase the possibility of observing a bias toward “same” responses in the 25% change condition. Second, to avoid any misunderstanding regarding the large color differences in our study, participants first engaged in a training block in which they received feedback after every response. Participants also received feedback at the end of every block.

Participants. A total of 73 participants took part in the experiment. We aimed for a total of 80 participants but did not reach that goal by a predefined date deadline. This resulted in 36 participants in the 75% change trial condition and 37 participants in the 25% change trial condition. The average age of our participants

⁸ The preregistration is available on the OSF (<https://osf.io/4gh9e>).

was 24.5 years, ranging from 18 to 35 ($SD = 3.90$). In exchange for their participation, they received CHF 15 or course credit. Experimental sessions took about 50 min. All participants reported having normal or corrected-to-normal vision and normal color vision.

Procedure. The experiment started with a practice block of 40 trials in which only binary responses were requested, followed by 10 test blocks, also with 40 trials, in which responses were given using a confidence-rating scale. In the practice block, half of the trials were change trials; Also, participants received feedback after each trial in the form of a green checkmark “✓” (correct response) or a red “×” (incorrect), presented at the center of the screen. In the remaining blocks, the proportion of change trials were either 75% or 25%, depending on the participant’s condition. In contrast with Experiment 1, this proportion stayed the same throughout the experiment (i.e., the proportion of change trials was manipulated between subjects). Participants did not receive feedback after each trial anymore. However, after each block, they were reminded of the actual proportion of change trials in the experiment together with their proportion of “change” responses in that block (see Dube & Rotello, 2012).

Results

In conditions with 75% and 25% change trials, the average proportion of “change” responses was .77 and .34, respectively. In terms of response-bias measure c , we found a (weak) bias toward “same” responses in the 25% change condition ($M = -0.08$, $SD = 0.34$, $W = 482$, $p = .05$) and a (stronger) bias toward “change” responses in the 75% change condition ($M = 0.47$, $SD = 0.36$, $W = 3$, $p < .0001$).⁹ Figure 4 shows the ROC functions obtained with the grouped confidence ratings from both bias conditions. Once again, they show the expected curvilinear, symmetrical shape.

As shown in Figure 5 (lower row, right panels) we again find evidence for biased confidence judgments in the 75% condition. The observed proportions of minimum-confidence “change” responses were generally greater in same trials, indicating the presence of a bias that is not permitted under a discrete-state model. Specifically, $P(\text{“change}_{\min}” | \text{Same trial})$ was greater/equal/smaller than $P(\text{“change}_{\min}” | \text{Change trial})$ in 75%, 5%, and 20% of the individual cases, respectively. At the group level, $P(\text{“change}_{\min}” | \text{Same trial})$ and $P(\text{“change}_{\min}” | \text{Change trial})$ were .12 and .09, respectively.

The individual minimum-confidence responses in the 75% condition were fitted with the same joint-binomial hierarchical-Bayesian model used in Experiment 1. As a reminder, the discrete-state account expects the difference parameter μ_{δ} to be greater or equal to zero. Parameter estimates yielded a posterior mean of μ_{δ} of -0.14 , with a 95% credibility interval of $[-0.20, -0.08]$ (see Figure 5). We obtained a BF of roughly 30,000 (as only two of the 60,000 μ_{δ} samples were larger than zero), indicating again strong support for the presence of biased confidence ratings. This result was again corroborated by means of posterior predictive tests. For the model that is constrained to follow the predictions of the discrete-state account, we obtained a Bayesian p value of .002. For the model that can produce biased confidence judgments, we obtained a Bayesian p value of .40.

Establishing the Scope of the Critical Test

At this point, it is not clear whether the implications of our test results are circumscribed to an overly simplistic set of models, especially in the case of the discrete-state model. This discussion is especially relevant given that both of our studies only used five-item arrays, a number that does not surpass many people’s working memory capacity (Cowan, 2001). This means that processes other than guessing are playing a major role. The purpose of this section is to provide some clarification on this matter and show that the model predictions discussed above hold across several possible extensions and/or modifications. We also discuss one discrete-state model variant that can account for biased confidence judgments and show that it outperforms a “pure” continuous counterpart.

Adopting Non-Gaussian Distributions in the Continuous Model

The illustration of the continuous model given in Figure 2 assumes that memory-strength distributions are Gaussian with equal variance. It is reasonable to ask whether the ability to predict biased confidence ratings requires specific distributional assumptions. It does not. The possibility of biased confidence only requires the ability to establish a pair of confidence criteria along an interval of latent values in which one stimulus type is more likely than the other. Note that differences in likelihood are implied by the mere observation of above-chance performance, as it implies that values larger than the binary criterion τ are more likely under same trials, and values below τ are more likely under change trials.¹⁰

Relaxing the Discrete-State Model’s Confidence Mapping

The discrete-state model can account for the observed biased confidence judgments if we relax the way in which each discrete state is mapped onto the confidence scale (e.g., Malmberg, 2002). So far, we have assumed that the memory state reached with probability m is always mapped onto the levels of the confidence-rating scale associated with one of two binary responses (see also Kellen, Singmann, Vogt, & Klauer, 2015; Klauer & Kellen, 2010, 2015; Province & Rouder, 2012). Note that by doing so we are not introducing any additional constraints to the model—we are merely retaining the assumption made in the case of binary choices.

We do not see this extension as a convincing way to salvage discrete-state models. Specifically, we do not find any justification

⁹ Given that the bias towards “same” responses in the 25% change condition is relatively weak, making the observation of biased confidence judgments unlikely (see also Footnote 5). And indeed, no bias was found. We decided to omit these analyses from the main text and report them in the OSF project. In any case, we report the relevant data in Figure 5.

¹⁰ Note that the assumption that ROC functions are concave (which includes linearity as a boundary case) implies that the likelihood ratio is monotonic. Any point of the ROC with slope larger/smaller than 1 indicates that the value of binary-response criterion τ is more likely under the latent distribution associated with same/change trials (for details, see Kellen, Winiger, Dunn, & Singmann, 2019). To obtain biased confidence ratings, one only needs to place the confidence criteria associated with a “change”/“same” response along a range of values in which the latent memory-strength values are more likely under the distribution associated with same/change trials.

for not imposing the constraints already in place when dealing with binary choices. Also, such a relaxation implies that individuals are willing to go against their memories in order to respond in conformity with the biases promoted by the experimental manipulation (e.g., responding “same” in a block with 25% change trials despite detecting a change), while simultaneously being willing to respond counter to the same biases when *guessing* (e.g., guessing “change” in a block with 25% same trials). Such an account is directly at odds with the idea that response biases affect first and foremost guesses (see Erdfelder, Küpper-Tetzl, & Mattern, 2011; Krantz, 1969; Luce, 1963). Moreover, we note that by assuming a relaxed mapping, proponents of discrete-state models can no longer resort to ROC shapes as a source of empirical support, given that the models no longer make clear predictions.¹¹

Swap Errors in the Discrete-State Model

One limitation of our discrete-state model is that it omits the possibility of participants incorrectly associating one of the other studied colors with the test position—a *swap error* (Bays, Catalao, & Husain, 2009; Wheeler & Treisman, 2002). Let w denote the conditional probability of a swap error, given that the correct color was not remembered. In the case of same trials, swap errors will always lead to a “change” response, as the color has to be different than the one presented at test. In contrast, in change trials, the probability of the remembered color mismatching the one at test is $\frac{3}{4}$ (see Donkin, Tran, & Le Pelley, 2015). The equations for minimum-confidence “change” judgment are then:

$$\begin{aligned} P(\text{“change}_{\min}” | \text{Change trial}) &= m \cdot \xi_{\min} + (1 - m) \cdot w \cdot \frac{3}{4} \cdot \xi_{\min} \\ &\quad + (1 - m) \cdot (1 - w) \cdot (1 - g) \cdot \gamma_{\min}, \\ P(\text{“change}_{\min}” | \text{Same trial}) &= (1 - m) \cdot w \cdot \xi_{\min} \\ &\quad + (1 - m) \cdot (1 - w) \cdot (1 - g) \cdot \gamma_{\min}. \end{aligned}$$

Based on these equations, we can see that a biased confidence rating is only expected when $\frac{m}{(1-m)} < \frac{w}{4}$, which only holds when $m \leq .20$. The upper boundary $m = .20$ requires that $w = 1$, which implies that there are no guessing-based responses, only swap errors. The required m values are extremely low—with a set size of five items, they would imply a discrete capacity (with successful binding) of at most one item. Such a characterization is not consistent with the overall performance observed in both experiments (see the ROC plots in Figure 5).

Color-Position Binding Failures

Now, let us consider an extended model according to which a person can also make a judgment based on the fact that they remember the color presented at test from earlier in the trial but not its exact location:¹²

$$\begin{aligned} P(\text{“change}_{\min}” | \text{Change trial}) &= m \cdot \xi_{\min} + (1 - m) \cdot c \cdot z \cdot \omega_{\min} \\ &\quad + (1 - m) \cdot (1 - c) \cdot (1 - g) \cdot \gamma_{\min}, \\ P(\text{“change}_{\min}” | \text{Same trial}) &= (1 - m) \cdot c \cdot z \cdot \omega_{\min} \\ &\quad + (1 - m) \cdot (1 - c) \cdot (1 - g) \cdot \gamma_{\min}, \end{aligned}$$

with c denoting the probability that the color at test is remembered (but not its location), z as the probability that this color is

not attributed to the test item location, and ω_{\min} corresponding to the probability of mapping this judgment onto a minimum-confidence “change” response. Once again, it is easy to see that $P(\text{“change}_{\min}” | \text{Change trial})$ cannot be smaller than $P(\text{“change}_{\min}” | \text{Same trial})$, therefore excluding the possibility of biased confidence.

Assuming Varying Discrete-State Probabilities

One way to accommodate biased confidence ratings is to allow for probability m to differ between change and same trials (i.e., establish m_{change} and m_{same} parameters instead of a single m parameter). For instance, the confidence bias observed in the 75% change condition would be accounted for if $m_{\text{change}} > m_{\text{same}}$. One explanation for these probability differences is that the differential expectations induced by base-rate manipulation affects sensory processing (e.g., Summerfield & de Lange, 2014). Although we cannot provide a clear-cut evaluation of this hypothesis using the present data, we can nevertheless point out that it yields predictions that do not seem to pan out in when inspecting the binary-response ROC data coming from change-detection tasks. Specifically, the discrete-state model no longer expects linear and symmetric binary-response ROCs when m_{change} and m_{same} are allowed to differ across response-bias conditions. To illustrate this point, we considered three possibilities:

1. m_{change} and m_{same} vary in opposite directions across response-bias conditions, such that $m_{\text{change}} > m_{\text{same}}$ when there is a bias toward “change” responses and $m_{\text{change}} < m_{\text{same}}$ when there is a bias toward “same” responses.
2. m_{change} varies across response-bias conditions, but m_{same} remains fixed (the inequalities stated above are also expected).
3. m_{same} varies across response-bias conditions, but m_{change} remains fixed (the inequalities stated above are also expected).

As shown in the left panel of Figure 6, the first possibility results in curved binary-response ROCs, which mimic the predictions of the continuous model illustrated in Figure 3. The prediction is somewhat implausible given that binary-response ROCs obtained in change-detection paradigms are generally well captured by a linear and symmetric function (e.g., Donkin et al., 2014; Rouder et al., 2008). If this first possibility held, then one would expect the extant ROC data to generally favor the continuous model. In turn, the center and right panels of Figure 6 show that the second and third possibilities would lead to asymmetric binary-response ROCs. These asymmetries are at odds with the symmetric functions predicted by both discrete-state and continuous models (see Figure 3). They are also somewhat implausible given that to the best of our knowledge, ROC data coming from change-detection

¹¹ It is worth pointing out that Malmberg’s (2002) case for a relaxed mapping of memory states hinges on the imposition of additional constraints on the mapping of guesses. Specifically, Malmberg assumed that guesses are distributed uniformly across confidence-rating scale. Without this constraint, one no longer needs to relax the mapping of memory-based responses to predict curvilinear ROCs (see Kellen & Klauer, 2015; Klauer & Kellen, 2010).

¹² As a reminder, note that the color tested in change trials was always presented in the study array of the same trial in one of the other locations.

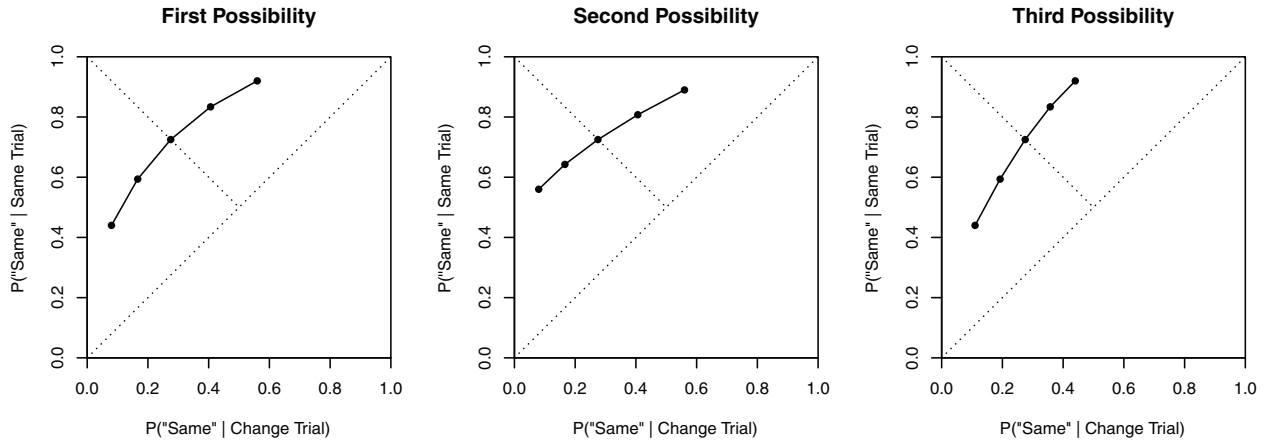


Figure 6. Illustrations of binary-response receiver operating characteristics (ROCs) for three possibilities of varying parameters m_{change} and m_{same} . In the left panel, the parameter values are $m_{\text{change}} = [.600, .525, .450, .375, .300]$, $m_{\text{same}} = [.300, .375, .450, .525, .600]$, and $g = [.20, .35, .50, .65, .80]$, with each entry corresponding to one response-bias condition, going from a bias toward “change” to a bias toward “same.” In the center panel, m_{same} is fixed to .45, whereas in the right panel, m_{change} is fixed to .45.

tasks have generally been well accounted for by models assuming symmetrical functions (whether they are continuous or discrete). One counterargument is that it is entirely possible for a small asymmetry to be masked by the noise usually found in binary-response ROC data (for a review, see Kellen, Klauer, & Bröder, 2013). But then again, this noise would have to somehow leave the prediction of biased confidence ratings unaffected.

Allowing for Fallible Memory Representations in the Discrete-State Model

Our discrete-state model assumes that items stored in VWM are represented with high fidelity, such that responses based on them are always accurate. It is not difficult to see how this assumption can be unreasonable in certain circumstances. For example, it implies the ability to detect the smallest differences between colors. Our first reaction to such criticism is to note that both experiments reported here relied on distinct colors, which should reduce the probability of memory-based errors. Moreover, to dismiss any concern regarding the possibility of individuals misunderstanding the large difference between the colors, participants in Experiment 2 were given trial-by-trial feedback in an initial training phase.

But the key question remains: Could a somewhat fallible representation provide the discrete-state model with the ability to account for biased confidence judgments? The answer is *no*. To show this, we have to extend the model so that it includes the possibility of incorrectly remembering the color at a given position, while preserving the conditional-independence assumption (Rouder & Morey, 2009). Specifically, let m^* denote the probability that a remembered color in VWM is correctly determined to be different from the test item in a change trial. Also, let m^{**} denote the probability that a remembered color in VWM is incorrectly determined to be different than the test item in a same trial. Based on this extension, the equations for minimum-confidence “change” judgment correspond to

$$\begin{aligned} P(\text{“change}_{\min}” \mid \text{Change trial}) &= m \cdot m^* \cdot \xi_{\min} + (1 - m) \cdot (1 - g) \cdot \gamma_{\min}, \\ P(\text{“change}_{\min}” \mid \text{Same trial}) &= m \cdot m^{**} \cdot \xi_{\min} + (1 - m) \cdot (1 - g) \cdot \gamma_{\min}. \end{aligned}$$

It follows that biased minimum-confidence judgments are only expected when the probability of an accurate memory representation in VWM in a change trial, m^* , is *smaller* than the probability m^{**} of an inaccurate memory representation in a same trial; that is, $\frac{m^*}{m^{**}} < 1$. What this means is that the memory representations provided by the discrete-state model would have to predict below-chance accuracy, which is not to be expected under any reasonable circumstances. To show this, let us consider a case in which both m^* and m^{**} are determined by the comparison of noisy representations and test items using a pair of perceptual thresholds. More specifically, let us assume that the noisy representation x of a given studied item is characterized by the normal distribution illustrated in Figure 7. When a test item T with value t is presented, the color is deemed to be the same as the one stored in VWM if x falls within the interval $[t - \Delta, t + \Delta]$, and different otherwise. The right panel of Figure 7 shows the ratio $\frac{m^*}{m^{**}}$ under different values of Δ and t . This ratio is larger than 1 across all the different values considered, therefore failing to show the condition necessary for the model to predict biased confidence judgments.

Casting Working-Memory Slots Through Mixture Signal-Detection Modeling

Finally, a discrete-state model extension that can account for biased confidence judgments in a plausible way assumes that people’s responses consist of a mixture of guesses and memory-based judgments, the latter being based on a comparison between graded and fallible memory representations with response criteria (e.g., Nosofsky & Gold, 2018; Xie & Zhang, 2017; see also Keshvari, van den Berg, & Ma, 2013; Zhang & Luck, 2008). As illustrated in Figure 8, memory-based confidence judgments are based on a comparison

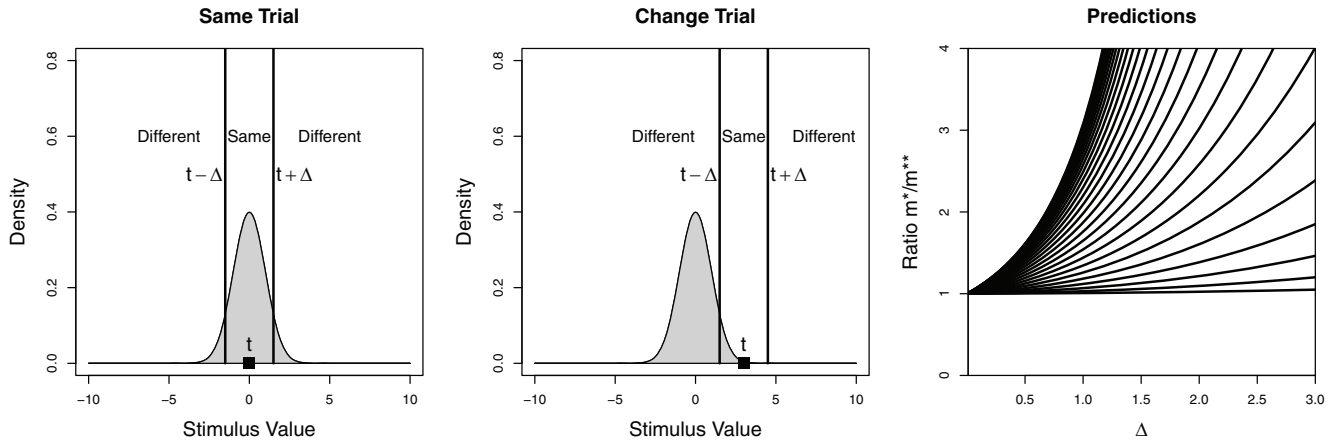


Figure 7. Example illustration of graded, fallible memory representation and its predictions. The densities characterize the fallible memory representations, whereas t corresponds to the value of the test item.

between the memory representation and the item presented at test. Specifically, the difference between the two is compared with pairs of confidence criteria. Confidence judgments are determined by the tightest pair of criteria that includes the difference. For instance, low-confidence “change” judgments (e.g., Rating 4 in Figure 8) are expected when the difference is relatively small.

Despite some minor differences, the characterization of memory-based responses given above ends up being pretty much the same as the continuous account’s (see Figure 2; see also DeCarlo, 2013). Therefore, it is reasonable to question whether it is even necessary to consider a mixture between guesses and memory-based judgments. In other words, would a pure continuous account be enough, given that the memory component in the mixture is what is doing the leg work? Although the present data do not provide us with the means to implement a critical test on the need for guessing-based confidence judgments, a tentative assessment can be obtained through parametric modeling. The approach we pursued here consisted of fitting a mixture model to individual ROC data coming from each base-rate condition (details can be found in the OSF project).¹³ Using Bayesian methods, we evaluated the posterior estimates of the mixture weight quantifying the proportion of memory-based judgments (and its complement the proportion of guesses). If guessing-based responses make a non-negligible contribution, then the posterior distributions of individuals’ mixture weights tend to be concentrated on values away from 1.¹⁴ The posterior estimates reported in Figure 9 show that even though there is considerable uncertainty, the mixture weights were often concentrated along values away from 1. This result suggests that guessing-based responses play a non-negligible role in describing the data (see also Nosofsky & Gold, 2018).¹⁵

Conclusion

The present work shows that several discrete-state models of VWM exclude the possibility of biased confidence ratings. In two experiments, we were able to observe biased “change” judgments, a behavioral signature that speaks directly against these models. These results provide us with a clear standard for evaluating the

sufficiency and necessity of certain theoretical features (e.g., the nature of memory representations in VWM). Subsequent analyses show that a mixture account assuming pure guesses along with graded memory representations provides a good account of the results (e.g., Nosofsky & Gold, 2018). Altogether, these results contribute to the behavioral foundation of mixture modeling in VWM. These results also have strong implications on the interpretation of previous model comparisons. For instance, it no longer seems reasonable to consider model comparisons supporting a discrete-state account, when such results are predicated on models that fail the present critical test (e.g., Donkin et al., 2014; Rouder et al., 2008).

More broadly, the present work demonstrates the potential of critical tests in the comparison of formal theories (Birbaum, 2011; Kellen & Klauer, 2014, 2015). Researchers should keep the advantages of this approach in mind: First, it often allows for the direct testing and dismissal of broad classes of models; that is, strong inference (Platt, 1964). Second, it shifts the focus away from global model-performance statistics, which can be somewhat opaque, and places it entirely on the specific behavioral patterns that have a clear diagnostic value. Third, it contributes to the development of a corpus of behavioral results that any candidate

¹³ The mixture model used assumes that the memory representation in same trials follows a Normal distribution with mean 0 and variance 1. In turn, the representation of change also follows a normal distribution, this time with mean μ and variance 1. The confidence criteria were free to vary (aside from their order constraints). Guessing-based confidence judgments is captured by an unconstrained multinomial distribution. Finally, the model has a mixture-weight parameter ω .

¹⁴ The mixture model considered here has more parameters (16) than there are degrees of freedom (15), which means that not all of its parameters are identifiable. The lack of identifiability can sometimes be a problem, even in the context of Bayesian parameter estimation (see Spektor & Kellen, 2018). Fortunately, the mixture-weight parameter that we are interested in is not severely compromised given that it plays a major role in establishing the model’s ability to describe data, a role that cannot be generally fulfilled by the other model parameters.

¹⁵ For reference, the rank-correlation of the median posterior mixture weights across base-rate conditions in Experiment 1 (a within-subjects manipulation) was 0.84.

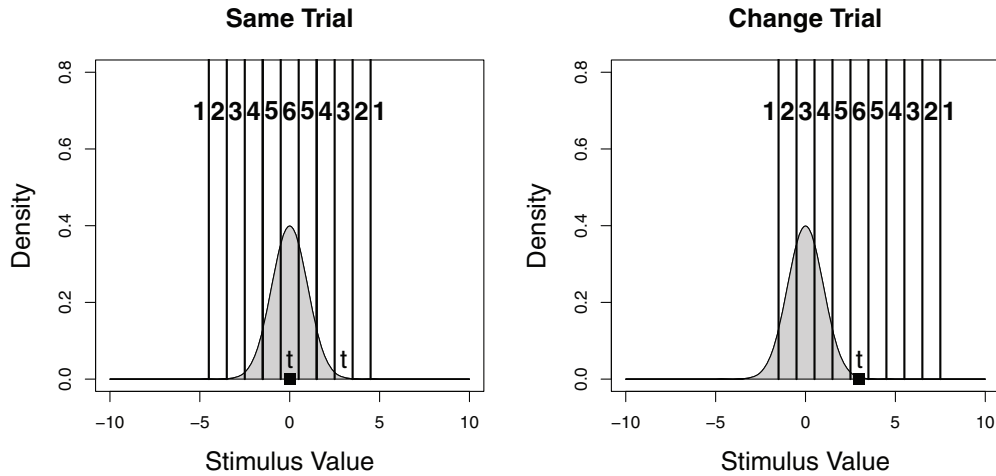


Figure 8. Example illustration of memory-based judgments according to the mixture model. The densities characterize the graded, fallible memory representations. The different vertical lines correspond to the confidence criteria, whereas t corresponds to the test item. For clarity, a six-point confidence scale is shown in the figure (rather than the eight-point confidence scale used in the experiment), which ranges from 1 (*very sure change*) to 6 (*very sure same*).

theory needs to be able to accommodate (see Oberauer et al., 2018).

To finalize, a clarification: The focus on specific portions of the data advocated by the critical-test approach discussed here can be seen as being antagonistic toward the development of accounts that

include as many sources of data as possible (e.g., categorical responses, RTs; Donkin et al., 2013). This is inaccurate. Both approaches are complementary, serving different goals and criteria (for a discussion, see Kellen, 2019): Critical tests provide sharp, localized evaluations of theories, whereas the development of

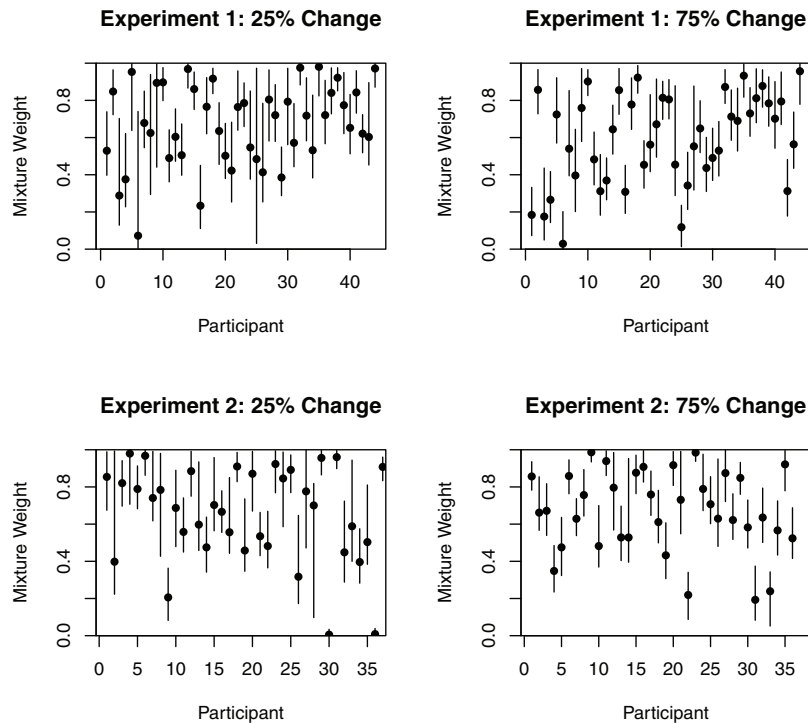


Figure 9. Median posterior estimates of the mixture weights and their respective 95% credibility intervals. A mixture weight of 1 would indicate that responses are solely determined by the memory component and there is no contribution from the guessing component.

increasingly encompassing models tests the ability of certain key theoretical notions to provide a competent and consistent characterization of the different types of data, and how these can differ across people and conditions. This complementarity is demonstrated by our parametric mixture-model fits, which show that among the theoretical accounts that happen to provide a plausible characterization of biased confidence judgments, one can find support for the presence of pure guesses.

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