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A Hierarchical Bayesian Signal Detection Model for Confidence Ratings

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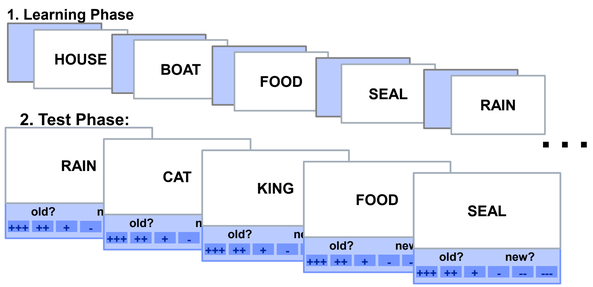
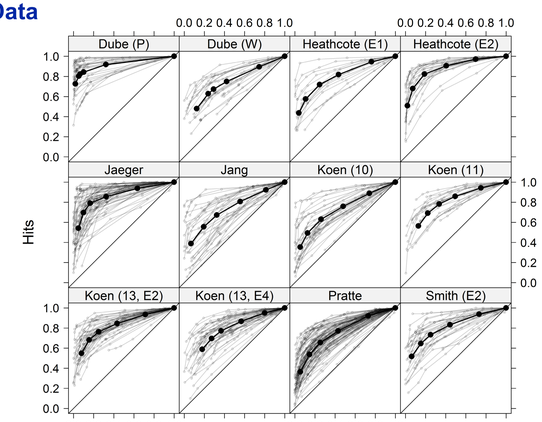
Task

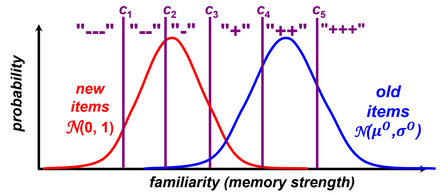


Data



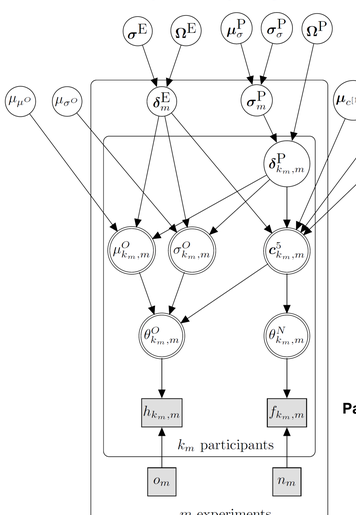
#	Source	N	trials/N
1	Dube & Rotello (2013, Exp. 1, Pictures)	27	400
2	Dube & Rotello (2013, Exp. 1, Words)	22	400
3	Heathcote et al. (2006, Exp. 1)	16	560
4	Heathcote et al. (2006, Exp. 2)	23	560
5	Jaeger et al. (2012, Exp. 1, no cue)	63	120
6	Jang et al. (2009)	33	140
7	Koen & Yonelinas (2010, pure study)	32	320
8	Koen & Yonelinas (2011)	20	600
9	Koen et al. (2013, Exp. 2, full attention)	48	200
10	Koen et al. (2013, Exp. 4, immediate test)	48	300
11	Pratte et al. (2010)	97	480
12	Smith & Duncan (2004, Exp. 2)	30	140
	Total N:	459	

Signal Detection Model



- **7 parameter unequal variance signal detection (UVSD) model**
- $\mu^N = 0, \sigma^N = 1, \mu^O, \sigma^O$, c_1, c_2, c_3, c_4, c_5

Hierarchical (2-level) Bayesian UVSD



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Model Structure

$$\mu^O_{k_m,m} = \mu_{\mu^O} + \delta^{\mathrm{P},[1]}_{k_m,m} + \delta^{\mathrm{E},[1]}_{m}$$
$$\sigma^O_{k_m,m} = \exp(\log(\mu_{\sigma^O}) + \delta^{\mathrm{P},[2]}_{k_m,m} + \delta^{\mathrm{E},[2]}_{m})$$
$$c^{[3]}_{k_m,m} = \mu_{c^{[3]}} + \delta^{\mathrm{P},[3]}_{k_m,m} + \delta^{\mathrm{E},[3]}_{m}$$
$$c^{[2]}_{k_m,m} = c^{[3]}_{k_m,m} - \exp(\log(\mu_{c^{[2]}}) + \delta^{\mathrm{P},[4]}_{k_m,m} + \delta^{\mathrm{E},[4]}_{m})$$
$$c^{[1]}_{k_m,m} = c^{[2]}_{k_m,m} - \exp(\log(\mu_{c^{[1]}}) + \delta^{\mathrm{P},[5]}_{k_m,m} + \delta^{\mathrm{E},[5]}_{m})$$
$$c^{[4]}_{k_m,m} = c^{[3]}_{k_m,m} + \exp(\log(\mu_{c^{[4]}}) + \delta^{\mathrm{P},[6]}_{k_m,m} + \delta^{\mathrm{E},[6]}_{m})$$
$$c^{[5]}_{k_m,m} = c^{[4]}_{k_m,m} + \exp(\log(\mu_{c^{[5]}}) + \delta^{\mathrm{P},[7]}_{k_m,m} + \delta^{\mathrm{E},[7]}_{m})$$

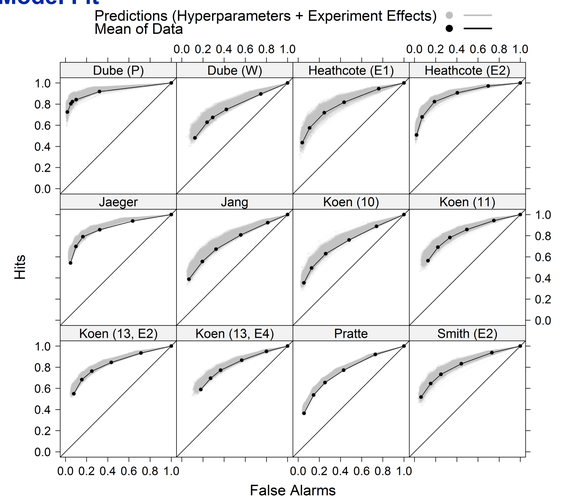
Data and Hyperparameters

$$h_{k_m,m} \sim \mathrm{Multinomial}(\theta^O_{k_m,m}, o_m)$$
$$f_{k_m,m} \sim \mathrm{Multinomial}(\theta^N_{k_m,m}, n_m)$$
$$\mu_{\mu^O} \sim \mathrm{Cauchy}(1, 4)$$
$$\mu_{\sigma^O} \sim \mathrm{Cauchy}^+(1, 4)$$
$$\mu_{c^3} \sim \mathrm{Cauchy}(0, 4)$$
$$\mu_{c^{[1,2]}} \sim \mathrm{Cauchy}^+(0.5, 2)$$
$$\mu_{c^{[4,5]}} \sim \mathrm{Cauchy}^+(0.5, 2)$$

Participant and Experiment Effect

$$\delta^{\mathrm{P}}_{k_m,m} \sim \mathrm{MvNormal}(\mathbf{0}, \Sigma_{\mathrm{P}})$$
$$\Sigma_{\mathrm{P}} = \sigma^{\mathrm{P}}_m \Omega^{\mathrm{P}}$$
$$\sigma^{\mathrm{P}}_m \sim \mathrm{Cauchy}^+(\mu^{\mathrm{P}}_\sigma, \sigma^{\mathrm{P}}_\sigma)$$
$$\delta^{\mathrm{E}}_m \sim \mathrm{MvNormal}(\mathbf{0}, \Sigma_{\mathrm{E}})$$
$$\Sigma_{\mathrm{E}} = \sigma^{\mathrm{E}} \Omega^{\mathrm{E}}$$
$$(\Omega^{\mathrm{P}}, \Omega^{\mathrm{E}}) \sim \mathrm{LKJ}(1)$$
$$(\mu^{\mathrm{P}}_\sigma, \sigma^{\mathrm{P}}_\sigma, \sigma^{\mathrm{E}}) \sim \mathrm{Cauchy}^+(0, 4)$$

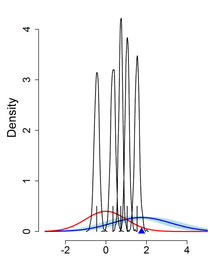
Model Fit



UVSD Hyperparameters

Param	Mean	2.5%	97.5%
μ^O	1.76	1.34	2.13
σ^O	1.44	1.35	1.53
c_1	-0.45	-0.70	-0.23
c_2	0.35	0.12	0.56
c_3	0.73	0.56	0.91
c_4	1.05	0.84	1.24
c_5	1.54	1.32	1.80

Note: Criteria, *c*, are on absolute familiarity scale (i.e., no increments)

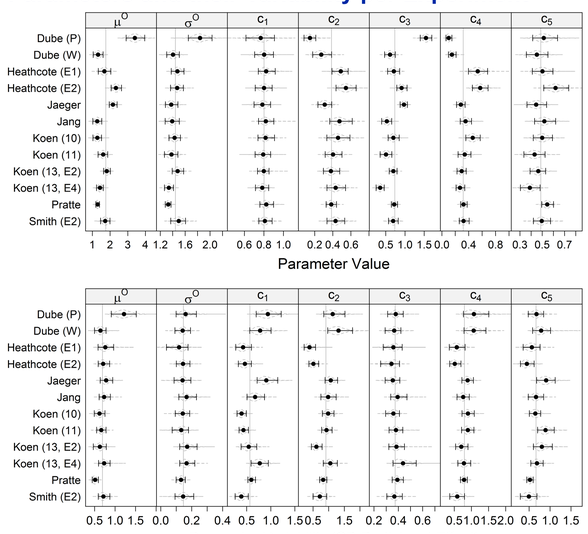


Parameter Correlations of Participant Effect

	σ^O	c_1	c_2	c_3	c_4	c_5
μ^O	**.72**			**.54**		
σ^O		-.15	**-.15**	**.66**	-.15	**-.37**
c_1			**.46**		**.42**	**.66**
c_2					**.83**	**.48**
c_3						**-.15**
c_4						**.61**

Notes: Correlations in bold (p < .05), other correlations (p < .1).
Correlations with criteria, *c*, are with increments with respect to c_3, c_2, or c_4.

Parameters and their Variability per Experiment



Note: Gray line shows mean of hyperparameter. Upper plots show hyperparameter plus experiment effect. Lower plots show SD of participants for given parameter.

Summary

- 459 participants from 12 experiments were jointly fitted with hierarchical Bayesian UVSD with random effects for participants and experiments.
- Model employed latent-trait approach (Klauer, 2010) with full variance-covariance matrix for all 7 UVSD parameters and each random effect.
- SD of participant effect was estimated individually for each study (with normal distribution as hyperparameter), but constant correlation matrix.
- Model provided good account and showed clear memory effect (μ^O = 1.76).
- Studying increased variability of familiarity distribution by > 40% (σ^O = 1.44), clearly larger then previously reported value of 1.25 (Ratcliff et al., 1992).
- Central criterion, c_3, located half-way between μ^N and μ^O.
- Outer criteria not symmetric: c_1 considerably further away from c_2, than c_5 away from c_4 (distance of c_2 and c_4 from c_3 approximately equal).
- On participant level, several robust correlations among parameters:
  - μ^O positively with σ^O and middle criteria c_3 (r > .5)
  - σ^O positively with middle criteria c_3 (r = .66) and negatively with distances of other criteria (r <= -.15)
  - Positively among all criteria distances (r > .45), very high for inner criteria from middle criteria (r > .8).
- Only study using pictures instead of words (Dube & Rotello, 2013, Exp. 1, Pictures) diverged most strongly from hyperparameters.
- Within-experiment parameter variabilities were not associated with magnitude of experiment effect (with exception of μ^O).