A meta-analysis of syllogistic reasoning using multinomial processing tree models

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A Syllogism

All frenchmen drink wine

Some wine drinkers are gourmets

Therefore, . . . ?

Types of Syllogism

- ► 4 types of moods
  All a are b
  Some a are b
  No a are b
  Some a are not b

- ► 2 assertions, 3 diff. terms

- ► 4 figures:
  Figure 1 (a-b b-c)
  Figure 2 (b-a c-b)
  Figure 3 (a-b c-b)
  Figure 4 (b-a b-c)

All b are a
Some b are not c
What follows?
... Some a are not c.

- ► 64 problems =
  4 (moods for 1st premise) x
  4 (moods for 2nd premise) x
  4 figures

- ► 9 possible responses =
  8 responses (Aac, Aca, ...)
  + "no valid conclusion"

Why syllogisms?

Isn't the market saturated?

- 2000+ years of Φ investigation
- 100+ years of Ψ research
- 100s of experiments
- (At least) 12 theories!

But:

- Lots of data
- Small set of problems
- Good testbed for cognitive science
- No theory has it right!

Theories of Reasoning: Mental Logic (Rips, 1994)

In a nutshell: Human reasoning follows the syntactic rules of formal logic (cp. Inhelder & Piaget, 1958):

▶ Application of (mental) inference rules
▶ Analogy to syntactic approaches
▶ Sequences of rules are a mental proof for conclusion

Mood	Rule	Meaning
A	All A are B	
	If $A(x)$ THEN $B(x)$	[If x is A then x is B]
	$A(a)$ and $B(a)$	[there are things, a, which are A and B]
	. . .	

x is a variable, a and b are "temporary names"

.. formal rules of logic ...

1. First-order predicate calculus
   (Rips, 1994; Braine & Rumain, 1983; Braine, 1998)
2. Verbal substitutions
   (Storring, 1908; Ford, 1995)
3. Monotonicity
   (Geurts, 2003; Politzer, 2007)

Mental Models (Johnson-Laird, 1983; 2006)

	Initial Model	Explicit Model ("fleshed out")
A: All A are B	[a] b [a] b ...	[a] [b] [a] [b] [¬a] [b] [¬a] [¬b]
I: Some A are B	a b a b ...	a b a b ¬a b a ¬b ¬a ¬b
E: No A are B	[a] [a] [b] [b] ...	[a] [¬b] [a] [¬b] [¬a] [b] [¬a] [b] ¬a ¬b

Notation:

- ► [] defines set of elements; none of elements appear elsewhere
- ► ¬ defines negation
- ► ... defines implicit information

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... of models and diagrams ...

1. Venn diagrams
   (Newell, 1981)
2. Euler circles
   (Erickson, 1974; Guyote & Sternberg, 1981; Ford, 1995)
3. Source founding
   (Stenning & Yule, 1997)
4. Verbal models
   (Polk & Newell, 1995)
5. Mental models
   (Johnson-Laird & Steedman, 1978; Bucciarelli & Johnson-Laird, 1999)

Probabilistic reasoning (Chater & Oaksford, 1999)

Probability heuristics model (PHM) is based on an informational ordering of the quantifiers: $A > I > E > O$

1. **Min-heuristic:** quantifier of the conclusion is the one of the least informative premise
2. **Max-heuristic:** Subject of the conclusion is end-term of the min-premise else it is the end-term of the max-premise
3. **p-entailment:** the next most preferred conclusion will be the p-entailment of the conclusion predicted by the min-heuristic

. . . heuristic theories . . .

1. Atmosphere heuristic
   (Woodworth & Sells, 1936; Begg & Denny, 1969; Revlis, 1975; Revlin et al., 1980)

2. Matching heuristic
   (Wetherick & Gilhooly, 1990)

3. Illicit conversion
   (Chapman & Chapman, 1959; Revlis, 1975)

4. Probability heuristics
   (Chater & Oaksford, 1999)

Meta-analysis of Khemlani & Johnson-Laird, 2012

- ▶ Only experiments with data for all 64 pairs of premises and participants formulated own conclusions
- ▶ Meta-analysis of 6 studies and 12 theories of syllogisms (Khemlani & Johnson-Laird, 2012).

How to assess a theory of syllogisms:

- ▶ # of hits
- ▶ # of correct rejections
- ▶ # of hits + correct rejections = correct predictions

Percentages of different types of response

	Aac	Eac	Iac	Oac	Aca	Eca	Ica	Oca	NVC
AA1	81	2	6	0	1	0	1	0	1

Meta-analysis of Khemlani & Johnson-Laird, 2012

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Responses that occur significantly often (binomial test, $p < .01$)

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MMT	Aac	Aca	Ica						

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Hits and correct rejections + False alarms

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Hits	=	1 (out of 1) =	100 %
Correct rejections	=	6 (out of 8) =	75%
Correct predictions	=	7 (out of 9) =	78%

Meta-analysis of Khemlani & Johnson-Laird, 2012: Predictive Power

Theory	Correct prediction (0 to 100)	Perfect predictions (0 to 100)
Atmosphere	78	11
Matching	71	6
Conversion	83	8
Probability heuristics	73	2
Logic (PSYCOP)	77	2
Verbal models	84	14
Mental models	78	25

Why a New Meta-Analysis?

Consider the syllogism:

$$\frac{\text{Some B are A}}{\text{Some C are B}}$$
$$\overline{\text{What follows?}}$$

- Mental Model Theory (Ordered, Dual-Process):
  - Initial Model (Type 1): Ica, Iac
  - Flesh-out (Type 2): NVC
- Probabilistic Approach (Ordered, no Dual-Process):
  - Ica, Oca
- Mental Logic (Not ordered, no Dual-Process):
  - NVC

This Meta-Analysis

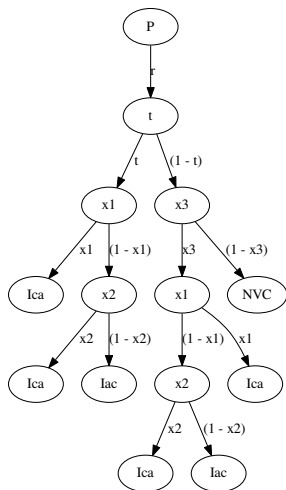Use multinomial processing tree (MPT) models for more fine grained predictions of the theories:

- ▶ If there are numerical differences between significant responses

- ▶ If the theory predicts a (weak) ordering on the conclusions

- ▶ If the theory takes different reasoning processes (e.g., Type 1 and 2)

- ▶ ... and the possibility of guessing into account

Theory predictions are compared by G^2 and Fisher Information Approximation (FIA)

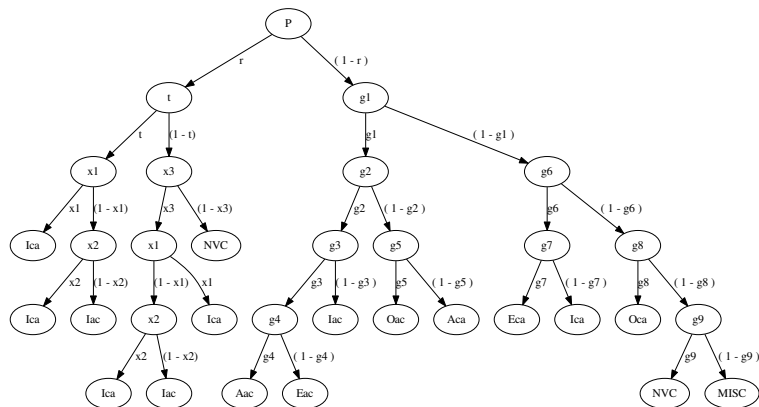# Multinomial Processing Tree Models: Guessing Tree
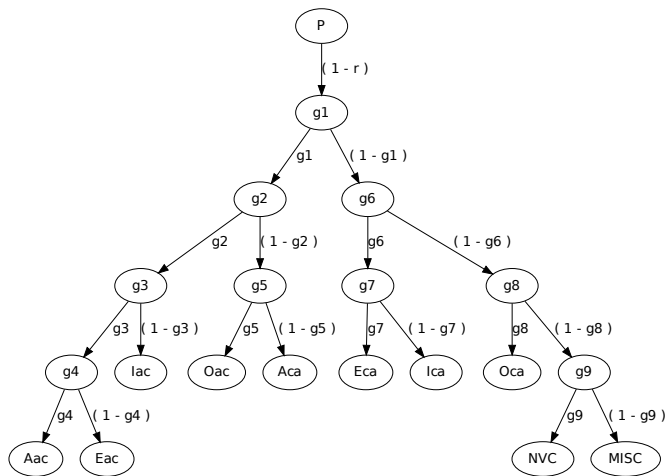
Multinomial Processing Tree Models: Reasoning

▶ Type 1 and Type 2 reasoning is represented by parameter $t \geq .5$

Multinomial Processing Tree Models: Complete

This Meta-Analysis: Syllogisms and Characteristics

- ▶ Raw data from the 6 studies of Khemlani & Johnson-Laird (2012) were used

- ▶ MPT models representing weak orders of predictions were fitted to the raw data (using MPTinR; Singmann & Kellen, 2013)

- ▶ Model fit combined with estimate of model complexity to select model that strikes best balance between fit and parsimony: Fisher Information approximation (FIA; Wu, Myung, & Batchelder, 2012)

The results fitting the aggregated studies

Theory	No. P.	G^2	FIA.penalty	ΔFIA	Rank
mReasoner	143	2474	294	0	1
Matching theory	201	2300	422	41	2
Atmosphere	137	3204	335	406	3
PHM	169	3903	342	763	4
Monotone	123	4661	279	1080	5
Verbal theory	129	6140	313	1853	6
Conversion	93	7413	243	2419	7
Grice+FOPC	132	8276	308	2915	8

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The reasoning parameter r in the MPT

The reasoning parameter r in percentage

	MMT	Match.	Verbal	PHM	Atm.	Mon.	FO	Conv.
Min	4	0	0	2	5	0	0	0
Max	89	89	88	88	89	88	88	87
Median	62	51	46	42	43	36	33	32
Mean	55	48	43	40	43	40	32	32
SD	24	24	23	22	25	28	26	26

- The reasoning parameter r describes the reasoning part
- e.g., MMT a median of 62% !

Summary and Conclusion

- ▶ Meta-study (KJ-L, 2012) shows a strong verbal reasoning theory
    - ▶ changes with our modeling
- ▶ Theories that integrate System 1 and System 2 reasoning provide better predictions
- ▶ Mental Model Theory in fact matches best, and
- ▶ ... assumes higher reasoning amount than other theories
- ▶ Multinomial Process Trees allows for model comparison
- ▶ and can be (even) used to build better theories!

The End

Thank You for Your Attention!

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